PAPER E

TCIMO | TERRY CHEW INSTITUTE OF MATHEMATICAL OLYMPIADS

SEAMO

Southeast Asian
Mathematical
Olympiad

2022

INTERMEDIATE

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED.

STUDENT'S NAME:

Read the instructions on the **ANSWER SHEET** and fill in your
NAME, SCHOOL and **OTHER INFORMATION**.
Use a 2B or B pencil.
Do **NOT** use a pen
Rub out any mistakes completely.

You **MUST** record your answers on the **ANSWER SHEET**.

Mark only **ONE** answer for each question.
Marks are **NOT** deducted for incorrect answers.

QUESTIONS 1 TO 20
Use the information provided to choose the **BEST** answer
from the five possible options.
On your **ANSWER SHEET** shade the option that matches
your answer.

QUESTIONS 21 TO 25
On your **ANSWER SHEET** write your answer within the box
provided. Units are not required.

You are **NOT** allowed to use a calculator.

1. Let a, b, c and d be integers such that $(a^2 + b^2)(c^2 + d^2) = 29$.

   Find the value of $a^2 + b^2 + c^2 + d^2$.

   (A) 28

   (B) 29

   (C) 30

   (D) 32

   (E) 34

2. Find the value of
   $$\frac{300^2}{253^2 - 247^2}$$

   (A) 30

   (B) 32

   (C) 34

   (D) 36

   (E) 40
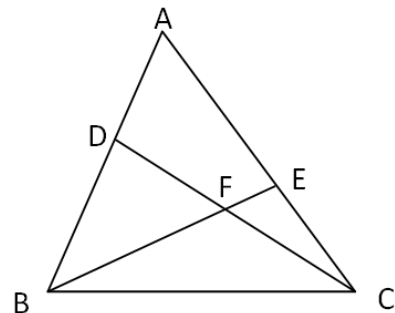
3. It is known that there is only one pair of positive integers a and b such that $a \le b$ and $a^2 + b^2 + 3ab = 719$.

   Find $(a + b)$.

   (A) 22

   (B) 24

   (C) 25

   (D) 26

   (E) 27

4. In an equilateral triangle $\triangle ABC$, D and E are two points on AB and AC, respectively. BE and CD intersect at F. It is known that $\angle BFC = 120°$.

   Which of the following is <u>TRUE</u> about the magnitudes of AD and CE?

   (A) $AD > CE$

   (B) $AD < CE$

   (C) $AD = CE$

   (D) $AD = \dfrac{5}{4} CE$

   (E) Impossible to tell

5. How many consecutive zeros are there at the end of the decimal representation of 80! ?

   (A) 16

   (B) 17

   (C) 18

   (D) 19

   (E) 20

6. Find the remainder when $6^{273} + 8^{273}$ is divided by 49.

   (A) 0

   (B) 1

   (C) 2

   (D) 3

   (E) 4

7. The infinite sequence

   1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 ...

   is obtained by writing the positive integers in order. What is the 2022nd digit in the sequence?

   (A) 0

   (B) 1

   (C) 2

   (D) 3

   (E) 4

8. Which of the following numbers is the smallest?

   (A) $\sqrt{290} - 12$

   (B) $5\sqrt{13} - 8$

   (C) $\dfrac{1}{29}$

   (D) $29 - 2\sqrt{210}$

   (E) $13 - 2\sqrt{42}$

9. Let $n!$ denote the product

$$n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

If $f(n)$ is denoted by

$$f(n) = \frac{\left(\frac{100}{3}\right)^n}{n!}$$

Find the value of positive integer n when $f(n)$ is the largest.

(A) 29

(B) 30

(C) 31

(D) 32

(E) 33

10. $\triangle ABC$ is an equilateral triangle. D is a point outside of $\triangle ABC$ such that $AC = AD$. Given that $\angle CDB = x$, find the value of x.

(A) 30°

(B) 34°

(C) 36°

(D) 40°

(E) 45°

QUESTIONS 11 TO 20 ARE WORTH 4 MARKS EACH

11. If $A = 40°$ and $B = 5°$, find the value of $(1 + \tan A)(1 + \tan B)$.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

12. If $a + b = 2$ and $a^2 + b^2 = 5$, find the value of $a^3 + b^3$.

(A) 9

(B) 10

(C) 11

(D) 12

(E) 13

13. Simplify

$$\sqrt{3+2\sqrt{2}} + \sqrt{3-2\sqrt{2}}$$

(A) $4\sqrt{3}$

(B) $2\sqrt{3}$

(C) $2\sqrt{2}$

(D) $2\sqrt{5}$

(E) $3\sqrt{2}$

14. If n and k are positive integers that satisfy

$$\frac{7}{13} < \frac{n}{n+k} < \frac{6}{11}$$

Find the smallest n that satisfies the inequality.

(A) 9

(B) 10

(C) 11

(D) 12

(E) 13

15. Write down the last four digits of 7^{128}.

(A) 3323

(B) 4703

(C) 5701

(D) 6801

(E) 7319

16. In a pack of 10 watches, 3 are known to be defective. If 2 watches are selected at random from the pack, what is the probability that at least one is defective?

(A) $\dfrac{2}{11}$

(B) $\dfrac{1}{13}$

(C) $\dfrac{3}{13}$

(D) $\dfrac{7}{15}$

(E) $\dfrac{8}{15}$

17. Evaluate $\tan 40° + 2\tan 10°$.

(A) $\tan 45°$

(B) $\tan 50°$

(C) $\tan 60°$

(D) $\cot 35°$

(E) $\cot 50°$

18. Evaluate $\log_5 \sqrt{5\sqrt{5\sqrt{5 \ldots}}}$

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

19. In a survey concerning the brands of coffee for consumers, it was found that 50% drink Brand A, 45% drink Brand B, 40% drink Brand C, 25% drink both brands A and B, 10% drink both brands B and C, 16% drink both brands A and C and 8% drink all 3 brands.

What percentage of the consumers surveyed do not drink coffee?

(A) 5

(B) 6

(C) 7

(D) 8

(E) 9

20. Simplify

$$\sqrt{\log_2 3 \times \log_2 12 \times \log_2 48 \times \log_2 192 + 16} \\ - \log_2 12 \times \log_2 48 + 10$$

(A) 5

(B) 6

(C) 7

(D) 8

(E) 9

21. In the figure below, AB and CD are parallel chords of a circle with centre O and radius r cm. It is given that $AB = 46$ cm, $CD = 18$ cm and $\angle AOB = 3 \times \angle COD$. Find the value of r.
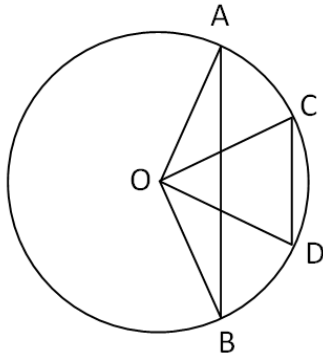

22. It is known that $S_n = u_1 + u_2 + u_3 + \cdots + u_n = 4^n - 1$. Find the 4th term of the sequence $[u_n]$.

23. Let $u_1, u_2, \ldots, u_{49}$ be an increasing arithmetic progression where the initial term is 25 and the difference between any successive terms is 3. Find the value of

$$\frac{3}{\sqrt{u_1} + \sqrt{u_2}} + \frac{3}{\sqrt{u_2} + \sqrt{u_3}} + \cdots + \frac{3}{\sqrt{u_{48}} + \sqrt{u_{49}}}$$

24. Two small identical circles of radii 5 cm and a big circle of radius 8 cm are embedded in rectangle $ABCD$. The circles are tangential to the rectangle. Given that $AB = 34 \, cm$, find the area of rectangle $ABCD$ in cm^2.
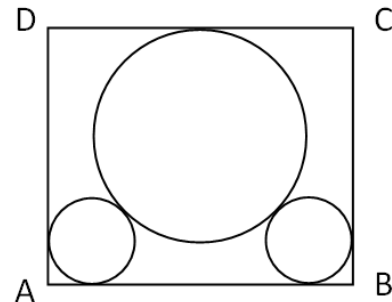
25. Given that

$$\sqrt{x} = \sqrt{23} - \frac{1}{\sqrt{23}}$$

find the value of

$$\frac{x + 2 + \sqrt{x^2 + 4x}}{x + 2 - \sqrt{x^2 + 4x}}$$

End of Paper

SEAMO 2022

Paper E – Answers

Multiple-Choice Questions

Questions 1 to 10 carry 3 marks each.

Q1	Q2	Q3	Q4	Q5
(C)	(A)	(B)	(C)	(D)

Q6	Q7	Q8	Q9	Q10
(A)	(A)	(D)	(E)	(A)

Questions 11 to 20 carry 4 marks each.

Q11	Q12	Q13	Q14	Q15
(B)	(C)	(C)	(E)	(D)

Q16	Q17	Q18	Q19	Q20
(E)	(B)	(A)	(D)	(B)

Free-Response Questions

Questions 21 to 25 carry 6 marks each.

21	22	23	24	25
27	192	8	612	529