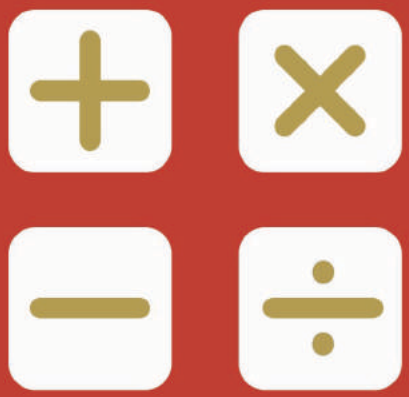
TCIMO | TERRY CHEW INSTITUTE OF MATHEMATICAL OLYMPIADS

PAPER F



SEAMO

Southeast Asian Mathematical Olympiad

2022

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED.

STUDENT'S NAME:

Read the instructions on the **ANSWER SHEET** and fill in your **NAME, SCHOOL** and **OTHER INFORMATION**.
Use a 2B or B pencil.
Do **NOT** use a pen
Rub out any mistakes completely.

You **MUST** record your answers on the **ANSWER SHEET**.

SENIOR

Mark only **ONE** answer for each question.
Marks are **NOT** deducted for incorrect answers.

QUESTIONS 1 TO 20
Use the information provided to choose the **BEST** answer from the five possible options.
On your **ANSWER SHEET** shade the option that matches your answer.

QUESTIONS 21 TO 25
On your **ANSWER SHEET** write your answer within the box provided. Units are not required.

You are **NOT** allowed to use a calculator.

QUESTIONS 1 TO 10 ARE WORTH 3 MARKS EACH

1. Find the smallest 4-digit number that has the same number of factors as 2022.

   (A) 1000

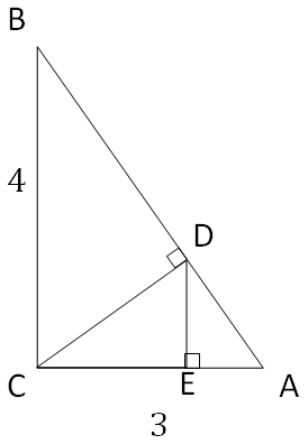
   (B) 1001

   (C) 1003

   (D) 1005

   (E) None of the above

2. In the triangle ABC, $\angle C = 90°$, $CA = 3$ and $CB = 4$. Point D is on segment AB such that CD is perpendicular to AB. Point E is on segment AC such that DE is perpendicular to CA. If the length of DE is $\frac{m^2}{n^2}$, where m and n are relatively prime, find $m+n$.



   (A) 3

   (B) 5

   (C) 7

   (D) 11

   (E) None of the above

3. If x and y are real numbers that satisfy $x^2 - 2xy + 2y^2 - 4y + 4 = 0$, find $x^4 + y^4$.

   (A) 2

   (B) 9

   (C) 64

   (D) 162

   (E) None of the above

4. Let O and I be the centers of the circumscribed circle and the inscribed circle of triangle ABC, respectively. Given that the sides of the triangles are 6, 8, and 10, and that the length of $OI = x$, find the value of x^2.

   (A) 2

   (B) 3

   (C) 4

   (D) 5

   (E) None of the above

5. Find the last 2 digits of 2022^{2022}.

(A) 22

(B) 04

(C) 36

(D) 84

(E) None of the above

6. Andrew and Bob are playing a game. There are 2022 cards in a pile, facing down. 1001 of them are black, and the rest are white.

Each round, 2 cards are taken consecutively. If the first card drawn is black and the second is white, Andrew wins the game. If the first card is white and the second is black, Bob wins the game. If the two cards are of the same color, the game continues to the next round without replacement of the cards.

If the probability of Andrew winning is x, find $\lfloor 2022x \rfloor$.

(A) 1

(B) 101

(C) 1001

(D) 1011

(E) None of the above

7. Let $\alpha = \frac{\pi}{18}$. Find the value of:

$$\cos\alpha + \cos 2\alpha + \cos 3\alpha + \cdots + \cos 18\alpha$$

(A) -1

(B) 0

(C) 1

(D) $\sqrt{3}$

(E) None of the above

8. Find the remainder when

$$20^{2027} + 2^{2027} + 23^{2027} + 32^{2027}$$

is divided by 2027.

(A) 0

(B) 27

(C) 77

(D) 207

(E) None of the above

9. Let $f(x) = (x+1)^2$. Find the sum of all values of x that fulfill the condition $fff(x) = x$.

(A) -8

(B) 8

(C) -1

(D) 1

(E) None of the above

10. In a triangle ABC, $AC = 2\sqrt{17}$, $BC = 8$, and $AB = 10$. Points D, E, and F are the midpoints of BC, CA, and AB, respectively. Points P and Q are the projections of points A and D on EF. Find the length of PQ.

(A) 1

(B) 2

(C) 3

(D) 4

(E) None of the above

QUESTIONS 11 TO 20 ARE WORTH 4 MARKS EACH

11. Find the number of ways to represent 2072 in the form

$$2072 = a \cdot 10^3 + b \cdot 10^2 + c \cdot 10^1 + d$$

where a, b, c, d are whole numbers and $0 \leq a, b, c, d \leq 99$.

(A) 208

(B) 200

(C) 168

(D) 188

(E) None of the above

12. $ABCD$ is a quadrilateral with $AB = BC = CD = DA$, and $\angle ABC < 90°$. Given that points E and F are on lines BC and CD, respectively, and $AB = AE = AF = FE$, find $\angle BEF$.

(A) 150°

(B) 140°

(C) 130°

(D) 120°

(E) None of the above

13. Any collection of n distinct integers between 1 and 20 has the property that there is always at least one pair of numbers such that one is a multiple of another.

Find the smallest number of n.

(A) 10

(B) 11

(C) 12

(D) 13

(E) None of the above

14. A three-digit number $\overline{abc}$ ($a, b,$ and c are not necessarily unique) does not have 0 as one of its digits. If $\overline{abc}$, $\overline{bca}$, and $\overline{cab}$ are all divisible by 7, find the sum of all possible $\overline{abc}$.

(A) 777

(B) 2331

(C) 7000

(D) 2100

(E) None of the above

15. In a triangle ABC with sides $a, b,$ and c, $a^2 + b^2 + c^2$ is 20 times the area of triangle ABC. Find the value of $\cot A + \cot B + \cot C$.

(A) 1

(B) 2

(C) 4

(D) 5

(E) None of the above

16. The probability of raining on a particular day is $\frac{3}{5}$. Andrew and Bella are going home from school. The probability of Andrew bringing an umbrella is $\frac{1}{3}$. The probability of Bella bringing an umbrella is $\boldsymbol{b}$. If it does not rain, the probability of them going back together is $\boldsymbol{c}$. If it rains, Andrew and Bella will go home together if and only if exactly one of them brings an umbrella, and the other does not. It is given that the probability of them going home together is $\frac{2}{5}$, and that the minimum value of $b^2 + c^2 = \delta$. What is the value of $\lfloor 100\delta \rfloor$?

(A) 20

(B) 25

(C) 33

(D) 40

(E) None of the above

17. Let

$$f(x) = \sqrt{\log_2 \frac{5x+1}{4x-1}}$$

If the function f is not defined for the domain $m < x \leq n$, what is the value of $\lfloor m + 100n \rfloor$?

(A) 2

(B) 20

(C) 23

(D) 27

(E) None of the above

18. The arrangements of the digits 0, 1, 2, 3, …, 8, 9 are said to be *'cheerful'* when the digits {0,1,2,3,4} are in ascending order and the digits {5,6,7,8,9} are in descending order. For example, 9801237465 is a *'cheerful'* number. If the first digit of the arrangement cannot be 0, how many *'cheerful'* arrangements are there?

(A) 126

(B) 252

(C) 1024

(D) 3024

(E) None of the above

19. The polynomial

$$1 - x + x^2 - x^3 + \cdots + x^8 - x^9$$

can be written in the form

$$a_0 y^9 + a_1 y^8 + a_2 y^7 + \ldots + a_8 y^1 + a_9$$

where $x = y - 1$. Find $|a_2|$.

(A) 1

(B) 9

(C) 36

(D) 45

(E) None of the above

20. Andrew and Bob are playing a game. There are 5 cards, numbered 1,2,3,4, and 5. Andrew shuffles the cards, and places them face-down on a table. In each round, Bob can choose any 3 cards on the table, and Andrew will state the value of one of the three cards chosen. Bob can continue to choose any 3 cards to ask Andrew as many times as he likes.

How many cards at most does Bob need to choose to ensure he knows the positions of all the cards?

(A) 5

(B) 4

(C) 3

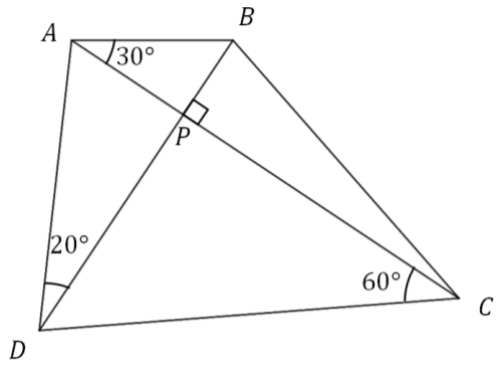
(D) Bob will not be able to know

(E) None of the above

QUESTIONS 21 TO 25 ARE WORTH 6 MARKS EACH

21. In a quadrilateral $ABCD$, AC is perpendicular to BD at P.

    If $\angle BAC = 30°$, $\angle ACD = 60°$, and $\angle ADB = 20°$, find $\angle ABC$ in degrees.



22. Let x_i be a real number such that x_i is a solution of the following equation:

$$\frac{64^x + 27^x}{36^x + 48^x} = \frac{13}{12}$$

What is the value of $100 \times \sum x_i^2$?

23. There are 21 balls numbered 1 to 21. The balls are all to be placed into 3 labeled boxes such that each box only contains either odd-numbered balls or even-numbered balls (a box can be empty). Find the number of ways to do so.

24. Let x and y be real integers such that

$$x^3 + y^3 = (x + y)^2$$

Let S be a set which contains all possible values of $(x + y)$. Find the sum of all elements in S.

25. Determine the number of ordered pairs of integers (m, n) for which $mn \geq 0$ and $m^3 + n^3 + 60\,mn = 20^3$.

End of Paper

SEAMO 2022

Paper F – Answers

Multiple-Choice Questions

Questions 1 to 10 carry 3 marks each.

Q1	Q2	Q3	Q4	Q5
(B)	(D)	(E)	(D)	(D)

Q6	Q7	Q8	Q9	Q10
(D)	(A)	(C)	(A)	(B)

Questions 11 to 20 carry 4 marks each.

Q11	Q12	Q13	Q14	Q15
(A)	(B)	(B)	(A)	(D)

Q16	Q17	Q18	Q19	Q20
(A)	(C)	(A)	(D)	(D)

Free-Response Questions

Questions 21 to 25 carry 6 marks each.

21	22	23	24	25
130	200	9210	8	22